S4A6 - Collaborative Formalization in Analysis

Seminar: summer semester 2024: Fridays 10 (c.t.)-12, room 0.011.
Goals of the course

- Collaborate on a formalization project
- Write high quality formalizations and review each other's work
- Help each other with questions and problems
- Collaborate on a blueprint
- Present about a topic in analysis, formalization or the ongoing project
- Incorporate some material into `mathlib`.

Presentation topics

The topics are ordered: preferably we get presentations about the top items of each list, the bottom items are optional.
Analysis topics: In your presentation carefully go through the proofs, and write up a blueprint for the formalization with detailed proofs that can be used as a basis for the formalization. The early topics go to students that are less well-versed in analysis. Participate in the formalization of these results, with the help from others.

- **[Stefan]** Introduction: Fourier transform on L^1 and L^2 and Plancherel theorem.
- Chapter 1 of [SS11] (L^p spaces and Banach spaces), especially focused on section 4 (the dual space of L^p).
- **[Matteo]** Real and complex interpolation [SS11, Sections 2.1-2.2] and [Tic23, Section 1.2]
- Applications of interpolation [SS11, Sections 2.4] and [Tic23, Section 1.3]
- Hardy spaces [SS11, Section 2.5 (& 2.3)] and [SM93, Chapter 3]
- BMO [SS11, Sections 2.6] and [SM93, Chapter 4]
- Abstract interpolation theory [Tic23, Sections 2.1-2.2]
- **[Tim]** Distributions [SS11, Sections 3.1-3.2]

Formalization topics: These are topics more focused on formalization. The first two topics ask you to describe a part of `Mathlib`, and you should explain how basic concepts are defined in the library and how to use them. The other topics are somewhat-related papers that you present. You participate in the formalization of one of the analysis topics.

- **[Nils]** Measure theory in `Mathlib`, including integration, L^p spaces and Haar measure (see [vD21]).

- Analysis in `Mathlib`: Banach spaces, Hilbert spaces, Fourier transforms.
- *A Formalization of the Change of Variables Formula for Integrals in mathlib* [Gou22]
- **[Kunhong]** *Multiple-Inheritance Hazards in Dependently-Typed Algebraic Hierarchies* [Wie23]
- *Aesop: White-box best-first proof search for lean* [LF23]
- *Formalizing the Divergence Theorem and the Cauchy Integral Formula in Lean* [Kud22]
- *Formalising the h-principle and sphere eversion* [vDMN23]

Project topics: Lead the formalization of these results, with the help from others. Near the end of the semester: present the progress made in the formalization of these results.

- L^p spaces, Fourier transforms and Plancherel theorem
- Interpolation theory
- Hardy spaces and BMO

Organizational remarks

- Since there is a formalization aspect to this course, there is a bit less focus on the talks. You therefore don't need to fill the full 90 minutes, and can give a 45-60 minute talk (but feel free to take the full 90 minutes if you have a lot of material to cover).
- Please make an account on the Lean Zulip: `https://leanprover.zulipchat.com/`. Send me an email with your username (you are encouraged to use your actual name for this). Feel free to introduce yourself or ask generic questions about Lean in *New Members*. The course discussion will happen in a private stream that I'll invite everyone to.
- I will make a Github repository for the formalization.

References

[Gou22] Sébastien Gouëzel, *A formalization of the change of variables formula for integrals in mathlib*, Intelligent Computer Mathematics (Cham) (Kevin Buzzard and Temur Kutsia, eds.), Springer International Publishing, 2022, pp. 3–18.

[Kud22] Yury Kudryashov, *Formalizing the Divergence Theorem and the Cauchy Integral Formula in Lean*, 13th International Conference on Interactive Theorem Proving (ITP 2022) (Dagstuhl, Germany) (June Andronick and Leonardo de Moura, eds.), Leibniz International Proceedings in Informatics (LIPIcs), vol. 237, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2022, pp. 23:1–23:19, doi:10.4230/LIPIcs.ITP.2022.23.

[LF23] Jannis Limperg and Asta Halkjær From, *Aesop: White-box best-first proof search for lean*, Proceedings of the 12th ACM SIGPLAN International Conference on Certified Programs and Proofs (New York, NY, USA), CPP 2023, Association for Computing Machinery, 2023, p. 253–266, doi:10.1145/3573105.3575671.

[SM93] Elias M Stein and Timothy S Murphy, *Harmonic analysis: real-variable methods, orthogonality, and oscillatory integrals*, vol. 3, Princeton University Press, 1993.

[SS11] Elias M Stein and Rami Shakarchi, *Fourier analysis: an introduction*, vol. 1, Princeton University Press, 2011.

[Tic23] Ian Tice, *A crash course in interpolation theory*, https://www.math.cmu.edu/~iantice/notes/interpolation_notes.pdf.

[vD21] Floris van Doorn, *Formalized Haar Measure*, 12th International Conference on Interactive Theorem Proving (ITP 2021) (Dagstuhl, Germany) (Liron Cohen and Cezary Kaliszyk, eds.), Leibniz International Proceedings in Informatics (LIPIcs), vol. 193, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2021, pp. 18:1–18:17, doi:10.4230/LIPIcs.ITP.2021.18.

[vDMN23] Floris van Doorn, Patrick Massot, and Oliver Nash, *Formalising the h-principle and sphere eversion*, Proceedings of the 12th ACM SIGPLAN International Conference on Certified Programs and Proofs (New York, NY, USA), CPP 2023, Association for Computing Machinery, 2023, p. 121–134, doi:10.1145/3573105.3575688.

[Wie23] Eric Wieser, *Multiple-inheritance hazards in dependently-typed algebraic hierarchies*, Intelligent Computer Mathematics (Cham) (Catherine Dubois and Manfred Kerber, eds.), Springer Nature Switzerland, 2023, pp. 222–236.